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DISCHARGE OF A TURBULENT JET IN A MONODISPERSE FLUIDIZED BED

N. N. Prokhorenko and Yu. I. Chernyaev

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The velocity of initial circulation is calculated and explanations are given for the mechanism of jet collapse and the existence of bubble and jet regimes.

Apparatus with fluidized beds of granular materials are widely used in industry, thanks to the high rate at which transport processes take place within them. In several processes, such as the granulation of solutions and melts in fluidized-bed apparatus, a turbulent jet inside the fluidized bed is used. A great many experimental studies have now been done on the development of the jet. These investigations are adequately detailed in [1]. Among the numerous effects seen in the interaction of the jet with the fluidized bed, the authors focused on the following: A dome-shaped gas jet is formed when a stream is discharged into a fluidized bed; when the ratio of the height of the jet to the height of the bed $x_f/H < 0.6$, the discharge takes place in the so-called "bubble" regime, accompanied by periodic collapse of the jet with the formation of bubbles; when the ratio $x_f/H > 0.6$, discharge occurs in the "jet" regime, with the boundaries of the jet being stationary and collapse of the jet occurring with greater frequency at the very edge of the nozzle. This can be seen quite clearly with the aid of high-speed photography.

We attempted to explain the physical mechanism of collapse of the gas jet and obtain quantitative relations establishing the conditions of this phenomenon.

The discharge of a stream into a bed of granular material was studied theoretically in [1, 2] and other works, and reliable results were obtained for the case of stationary beds. However, no explanation was found for the formation of the gas bubbles. It was only shown in [2] that "excessive constriction" of the stream, with the formation of bubbles, occurs at that area of the jet where the radial velocity component of the gas changes sign.

Below we present a method of analyzing the above phenomenon on the basis of a qualitative theory of differential equations describing the motion of particles of a granular material in a turbulent jet.

We will examine a circular, vertical, axisymmetric stream injected into a fluidized bed of particles of a narrow size fraction. For the sake of specificity in subsequent discussions, we will consider the boundaries of the gas jet (or the boundaries of the zone of discharge of the stream) to be the surface formed by the closest points to the stream axis at which the velocity of the gas is equal to the free-fall velocity of the particles at the corresponding porosity. We will also assume that the porosity and, thus the free-fall velocity

All-Union Scientific-Research and Planning-Design Institute for Assembly and Processing Lines, Moscow.

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of the particles under constrained conditions is the same over the entire boundary of the gas jet and that the gas velocity field within the boundaries is described by the following relations [1]:

$$v_x(x,\ 0,\ 0) = \begin{cases} \dfrac{2.73u_0r_0}{c_1x}, & \text{if} \quad x > \dfrac{2.73r_0}{c_1}, \\ u_0, & \text{if} \quad x \leqslant \dfrac{2.73r_0}{c_1}, \end{cases}$$

$$v_x(x,\ y,\ z) = v_0 + (v_x(x,\ 0,\ 0) - v_0)\left[1 - \frac{(y^2+z^2)^{0.75}}{b^{1.5}}\right]^2, \tag{1}$$

$$v_y(x,\ y,\ z) \approx 0, \quad v_z(x,\ y,\ z) \approx 0,$$

where x, y, and z are coordinates of a point of the jet in a Cartesian coordinate system with its origin at the center of the nozzle edge and the x axis directed along the jet axis inside the fluidized bed.

It should be noted that Eqs. (1) are valid for a stationary gas-velocity field in which the transverse components of the velocity vector are negligibly small compared to the longitudinal components.

From the aggregate of forces acting on a particle during its motion in the stream discharge zone, we will consider the force of gravity $\mathbf{F}_g$, buoyancy $\mathbf{F}_{Ar}$, drag $\mathbf{F}_{dr}$, and Magnus force $\mathbf{F}_M$, which arises due to particle rotation and nonuniformity of the gas-velocity field.

In vector form, the system of equations describing the motion of a particle in the stream discharge zone is:

$$\frac{d\mathbf{r}}{d\tau} = \mathbf{W}, \tag{2}$$

$$m\frac{d\mathbf{W}}{d\tau} = \mathbf{F}_g + \mathbf{F}_{Ar} + \mathbf{F}_{dr} + \mathbf{F}_M, \tag{3}$$

$$I\frac{d\mathbf{\Omega}}{d\tau} = \mathbf{M}, \tag{4}$$

where $\mathbf{r}$ is the radius vector of the center of the particle in the chosen coordinate system.

We will write the drag and Magnus force [3]:

$$\mathbf{F}_{dr} = \frac{1}{2}\rho\xi\frac{\pi d^2}{4}|\mathbf{V}-\mathbf{W}|(\mathbf{V}-\mathbf{W}), \tag{5}$$

$$\mathbf{F}_M = \frac{\pi d^3}{6}\rho\,[a(\mathbf{V}-\mathbf{W})\times(a\,\mathrm{rot}\mathbf{V} - 2\mathbf{\Omega})]. \tag{6}$$

Constraint of the particle motion is accounted for by the drag coefficient $\xi = k/(\varepsilon^m \mathrm{Re}^n)$ [4] and the coefficient $a = \varepsilon/(1-(1-\varepsilon)^{2/3})$ [5].

In connection with the considerable theoretical and empirical difficulties attendant to the determination of the moment $\mathbf{M}$ acting on the particle from the side of the gas flow, we will use the following model approach. We introduce a certain, yet-unknown sphere-surface mean thickness for the gas boundary layer immediately next to the particle surface and assume that within this layer the velocity of the gas changes from $\mathbf{V}(x,\ y,\ z)$ to the velocity at the surface, i.e., to $\mathbf{W}+[\mathbf{\Omega}\times\mathbf{R}]$, where $\mathbf{R}$ is the radius vector of a point of the particle surface relative to its center. We calculate the moment $\mathbf{M}$ as the sum of the moments of the frictional forces acting from the side of the gas on an element df of the particle surface:

$$\mathbf{M} = \int_F \left[\left(\frac{\rho\nu}{\Delta}(a(\mathbf{V}-\mathbf{W}) - [\mathbf{\Omega}\times\mathbf{R}])\right)\times\mathbf{R}\right]df = \frac{\pi d^4\rho\nu}{12\Delta}(a\,\mathrm{rot}\,\mathbf{V} - 2\mathbf{\Omega}). \tag{7}$$

After substitution of (5)-(7) into Eqs. (2)-(4), the system for the particles in the gas jet takes the form

$$\frac{dx}{d\tau} = w_x, \quad \frac{dy}{d\tau} = w_y, \quad \frac{dz}{d\tau} = w_z,$$

$$\frac{dw_x}{d\tau} = -g\,\frac{\rho_g - \rho}{\rho_g} + A\,|\mathbf{V} - \mathbf{W}|^{1-n}(v_x - w_x) + \frac{a\rho}{\rho_g}\left[w_y\left(a\frac{\partial v_x}{\partial y} + 2\omega_z\right) + \right.$$

$$\left. + w_z\left(a\frac{\partial v_x}{\partial z} - 2\omega_y\right)\right],$$

$$\frac{dw_y}{d\tau} = -A\,|\mathbf{V} - \mathbf{W}|^{1-n}\,w_y + \frac{a\rho}{\rho_g}\left[2w_z\omega_x + (v_x - w_x)\times\right.$$

$$\left.\times\left(a\frac{\partial v_x}{\partial y} + 2\omega_z\right)\right],$$

$$\frac{dw_z}{d\tau} = -A\,|\mathbf{V} - \mathbf{W}|^{1-n}\,w_z + \frac{a\rho}{\rho_g}\left[(v_x - w_x)\left(a\frac{\partial v_x}{\partial z} - 2\omega_y\right) - 2w_y\omega_x\right], \tag{8}$$

$$\frac{d\omega_x}{d\tau} = -2B\omega_x, \quad \frac{d\omega_y}{d\tau} = B\left(a\frac{\partial v_x}{\partial z} - 2\omega_y\right),$$

$$\frac{d\omega_z}{d\tau} = -B\left(a\frac{\partial v_x}{\partial y} + 2\omega_z\right),$$

where

$$A = 0.75\,\frac{\rho k \nu^n}{\varepsilon^m d^{1+n}\rho_g}, \quad B = \frac{5\nu\rho}{\rho_g d\Delta}.$$

In the first stage of our investigation we use the methods of the qualitative theory of differential equations to determine the values of the phase variables at which system (8) is in equilibrium, i.e., when the right sides of the equations vanish. This obviously occurs when

$$\omega_x = \omega_y = \omega_z = w_x = w_y = w_z = \frac{\partial v_x}{\partial y} = \frac{\partial v_x}{\partial z} = 0; \quad v_x = v_0, \tag{9}$$

where

$$v_0 = \left[\frac{g(\rho_g - \rho)}{A\rho_g}\right]^{1/(2-n)}.$$

The last equation in (9) means that, in the equilibrium state, the velocity of the gas is equal to the free-fall velocity of the particles under constrained conditions. This occurs at the boundary of the gas jet. The foregoing explains the physical significance of the boundary concept introduced above — the closest locus to the jet axis at which the forces acting on the particle are balanced.

In the second stage of our investigation we determine the character of the equilibrium points. This requires us to find the roots of the characteristic equation [6] of system (8). After some simple transformations, the characteristic equation can be represented in the form

$$\lambda^3(\lambda + Av_0^{1-n})(\lambda + 2B)^2(\lambda + (2-n)Av_0^{1-n})\left[\lambda^2 + \lambda(2B + Av_0^{1-n}) + 2ABv_0^{1-n} - \frac{a^2\rho}{\rho_g}\left(\frac{\partial^2 v_x}{\partial b^2}\right)_p v_0\right] = 0, \tag{10}$$

where $b = \sqrt{y^2 + z^2}$, with the index p denoting that the derivative is calculated at an equilibrium point. It is not hard to see that the discriminant of the expression in the square brackets is positive. Thus, all of the roots of the polynomial (10) are real. It can also be shown that if the following condition is satisfied

$$\frac{2AB}{v_0^n} - \frac{a^2\rho}{\rho_g}\left(\frac{\partial^2 v_x}{\partial b^2}\right)_p \geqslant 0, \tag{11}$$

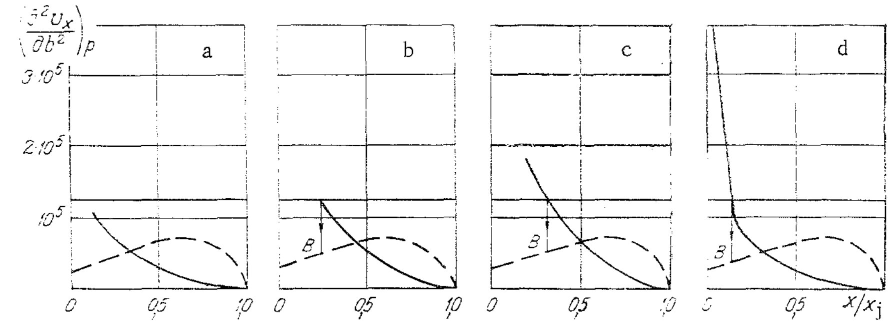

Fig. 1. Determination of coordinates of unstable equilibrium points: a) U_0 = 6.5 m/sec; b) U_0 = 7.16 m/sec; c) $U_0 = 5v_0$; d) $U_0 = 20v_0$. $(\partial^2 v_x/\partial b^2)_p$, $\mathrm{sec}^{-1}\cdot\mathrm{m}^{-1}$.

then there are no positive roots and the equilibrium is stable. If (11) is not satisfied, then there is one positive root and the equilibrium is unstable. As far as particle motion is concerned, this means that particles near points of the jet boundary where conditon (11) is satisfied will tend toward their equilibrium position. Conversely, particles located at boundary points where (11) is not satisfied will tend to abandon this position if there are any slight deviations.

Let us examine condition (11) in more detail and determine on which sections of the jet boundary it is satisfied and on which sections it is not. After simple transformations, (11) is changed to the form

$$\left(\frac{\partial^2 v_x}{\partial b^2}\right)_p \leqslant 7.5\, \frac{\xi \rho \nu}{a^2 d^2 \rho_g \Delta}\,. \tag{12}$$

The right side of (12) contains the hard-to-determine quantity Δ. However, it is significant that due to our assumption that the porosity of the bed is identical at the surface of the jet and that the gas is blown about all of the particles with the same velocity, the mean thickness of the boundary layer around each particle and, hence, the value of the right side of (12), are the same over the entire boundary of the jet. An estimate of Δ can be obtained by comparing (7) with the formula for the moment acting on a particle rotating in a quiescent fluid [3]:

$$-\frac{\pi d^4 \rho \nu}{12\Delta}\, 2\Omega = -\pi \rho \nu d^3 \Omega\,(1 + O(\mathrm{Re})). \tag{13}$$

From this we find

$$\Delta = \frac{d}{6(1 + O(\mathrm{Re}))}\,. \tag{14}$$

Having substituted the free-fall velocity of the particles into Re, we obtain $\Delta = 0.2\cdot10^{-6}$ m.

The left side of inequality (12) can be calculated exactly for the entire boundary of the jet, except for the initial section. The latter exception is due to the fact that there is no reliable data on the change in radial gas velocity for the initial section. Thus, all subsequent conclusions are valid for the main section of the jet.

Figure 1 shows the form of the jet boundary (dashed line), the value of the left side of inequality (12) (solid line), and the value of the right side (straight line). The values of the left and right sides were calculated for d = 2 mm, ρ_g = 1500 kg/m^3, r_0 = 3 mm, and ε = 0.8. The free-fall velocity when air is the fluidizing agent is v_0 = 5.51 m/sec for the foregoing conditions.

In Fig. 1a, inequality (12) is satisfied for the entire surface of the jet, i.e., particles over the entire boundary are in a stable equilibrium state and no particles are circulating through the jet.

Figure 1b shows unstable equilibrium point B, at which particles begin to enter the jet and circulate within it. It is important to note that the experimental value for the initiation of circulation obtained in [7] is 7.32 m/sec for the conditions being discussed, i.e., it agrees well with the value we have calculated.

Figure 1c also contains an unstable equilibrium point B at which the left side of (12) is equal to the right side. That part of the surface of the jet on which the particles are in unstable equilibrium is located below this point. The particles on the rest of the surface are in stable equilibrium. It is clear that particles must enter the jet on the section from the nozzle to point B. Calculations show that the distance from point B to the grate is slightly dependent on the stream exit velocity and is 7-8 nozzle radii. This fact has also been quantitatively confirmed by experiments [8, 9].

The fact that point B is an unstable equilibrium state means mathematically that any small change in the right side of the equations of system (8) will lead to bifurcation [10] of the phase portrait of system (8). Under the influence of small perturbations, particles of the granular material either leave the neighborhood of the unstable equilibrium point and approach the closest stable equilibrium point or undergo a slight movement (the unstable point is transformed into stable nearby points).

The first of these possibilities is a bifurcation of the phase portrait and takes the form of collapse of the jet. The frequency of this collapse is determined by the frequency of the turbulent pulsations of the gas-velocity field with a linear turbulence scale [11], large particle sizes, the porosity-dependent frequency of particle collision, etc.

Discharge of the stream occurs in Fig. 1d in a manner similar to that described above. The difference is that point B is located roughly in the middle of the jet in case c (with a low jet height) and at an elevation corresponding to 15% of the jet height in case d. Gas velocity on the jet axis at the site of the collapse exceeds the free-fall velocity of the particles by 0.5 m/sec in case c and by 15 m/sec in case d. This means that given a low jet height, after collapse the particles slowly gain the axial velocity and cover the jet cross-section, forming a closed cavity — a "bubble." When the jet is high, particles are rapidly carried out by the jet after collapse, and no bubble is formed. Visually, the bubble regime of discharge is seen with a low jet, and the jet regime is seen with a high jet.

NOTATION

x_j, H, height of jet and bed; v_0, u_0, free-fall velocity of particle under constrained conditions, nozzle exit velocity; $\mathbf{V}$, $\mathbf{W}$, v_x, v_y, v_z, w_x, w_y, w_z, velocity vectors of gas and particle and components of these vectors; Ω, ω_x, ω_y ω_z, angular-velocity vector of particle, components of same; r_0, b, radii of nozzle and jet; d, ρ_g, m, I, diameter, density, mass, and moment of inertia of particle; $\mathbf{F}_g$, $\mathbf{F}_{Ar}$, $\mathbf{F}_{dr}$, $\mathbf{F}_M$, and $\mathbf{M}$, forces of gravity, buoyancy, drag, Magnus force, and moment acting on a particle; τ, time; ρ, ν, density, kinematic viscosity of gas; ε, porosity (fraction of cavities) of the fluidized bed; k, m, n, coefficients dependent on the criterion $Re=|\mathbf{V}-\mathbf{W}|\,d/\nu$; λ root of the characteristic equation.

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MEASUREMENT OF THE STATISTICAL CHARACTERISTICS OF A TURBULENT BOUNDARY LAYER IN A NATURAL ENVIRONMENT

V. E. Aerov, A. D. Solodukhin,
S. V. Isaev, and N. N. Luchko

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Certain characteristics of a turbulent boundary layer are measured in a natural setting by means of an automated measurement system. Deviations of the investigated process from the Gaussian description are inferred on the basis of measurements of the higher statistical moments.

The work reported here was carried out as part of ebb-and-flow tidal studies during February-May, 1980. The measurements were performed at depths less than or equal to 100 m. The experimental test site was a level area of sufficiently large extent with a slightly varying diurnal current. The mean temperature at such shallow depths is determined mainly by solar radiation and heat transfer between the surface of the ocean and the atmosphere. The diurnal temperature variation falls within 1.5-2°C. The temperature drops at depths to 100 m do not exceed 0.1-0.3°C.

The salinity at the indicated depths varies within 2-3% limits. A maximum occurs at roughly 50 m. Diurnal variations were not observed. It can therefore be concluded that the salinity is virtually unaffected by the temperature in the top surface layer of the ocean. The majority of the Soviet and foreign measurement systems designed for the investigation of oceans and inland seas record the average parameters of the ocean medium. However, from the point of view of predicting and observing such phenomena as hurricanes, typhoons, tsunamis, etc., it is exceedingly important to understand the formation and evolution of small- and large-scale oceanic structures.

The AIK-1 measurement system includes primary sensors with capabilities for local measurements of the mean values of the velocity, temperature, and electrical conductivity as well as velocity and temperature fluctuations. The system can also be used for statistical processing of the experimental data in the real-time and data-storage modes.

A prototype model of the AIK-1 system was built by the Institute of Heat and Mass Transfer of the Academy of Sciences of the Belorussian SSR and the State University in Donetsk.

The AIK-1 automated measurement system is designed to record signals proportional to the local average hydrothermodynamic and small-scale fluctuations characteristics of the ocean medium at depths to 200 m, convert those signals into digital code, and enter the latter into a D3-28 minicomputer for on-line statistical processing and storage.

The following units are included in the automated measurement system:

a) a temperature-compensating hot-wire anemometer HA and resistance thermometer RT for simultaneously measuring the instantaneous values of the velocity and temperature fields in nonisothermal flow;

b) a conduction anemometer CA for measuring velocity fluctuations in a local volume of a slightly conducting fluid flow;

c) an electrical conductivity meter ECM for measuring the average value of the electrical conductivity;

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